Det Kgl. Danske Videnskabernes Selskab.
Biologiske Meddelelser. **I**, 1.

LAWS OF MUSCULAR ACTION

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K. KROMAN

KØBENHAVN
HOVEDKOMMISSIONÆR: ANDR. FRED. HØST & SØN, KGL. HOF-BOGHANDEL
BIANCO LUNOS BOGTRYKKERI
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I.

What is easiest, up hill or down hill? What is easiest, to lift yourself in the cross bar or to lower yourself again, all with the same constant speed?

These and similar questions have been raised over and over again by the theoretical sportsman, but a distinct and reliable answer has not yet appeared. At first thought one would perhaps be inclined to declare the uphill walk and the raising in the bar to be much more difficult than the downhill walk and the lowering; but, when we consider that the last named motions must be made at exactly the same rate as the first named, and that it is neither allowed to hurry down the hill nor to let oneself drop down in the bar, the uncertainty shows itself again, and, if the force exerted in the two kinds of motion is examined, one is rather inclined to declare that they must surely be equally difficult; nay, when very tired, you will easily, by trying, imagine that going downhill is after all more difficult than going up hill, at the same time readily admitting that you get more out of breath by walking uphill.

Nor, if you turn to physiology, will you get any exhaustive answer. The physiologists have certainly found a lot of special results; but as a rule they stand without any mutual connection. Sometimes they even contradict each other, and many of them only apply to the excised and dying muscles of frogs and other lower animals. Quite true, more than one hypothesis has been brought forward relating to the last cause of the contraction of muscles, but none of these hypotheses has advanced beyond the hypothetical stage, and even if we give up finding out *the reason why* the muscle acts in such a way, and are quite satisfied to know the *laws according to which* it acts thus, this demand is already a large step beyond what can at present be mastered. The question regarding muscular action is a very involved and unmanageable question.

The following remarks are therefore not intended as an attempt to carry our knowledge this step forward. That task will yet demand numerous investigations. It is only a small preliminarily contribution to the elucidation of the question I am putting forward, in as much as I intend to attack it from a point of view from which it has not up to now, to my knowledge, been examined, but from which, so it seems to me, a small, initiatory step forward might well be obtained.

When *the Law of the Conservation of Energy* was first formulated in a clear, precise and comprehensive way by H. HELMHOLTZ, 1847, the proposition was also at once put in its proper place: It is not an axiom i. e. a proposition which the nature of our thought compels us to accept. On the contrary, we can very well imagine a world where the proposition does not hold good. Nobody can establish a decisive proof of the impossibility of the perpetuum mobile. The proposition of the constancy of energy is an empirical proposition, but a proposition which innumerable experiences lead us to accept. It is an exceedingly probable proposition, and without doubt it holds good not only for the whole inorganic world, but also for each and all of the organisms. Helmholtz was likewise the first, or at least one of the first, who tried to show this by investigating and calculating the change of matter, and now a days hardly a single physiologist doubts the validity of the proposition. We are therefore justified in using it as a foundation for the following reflections.

The proposition of the conservation of energy has, however, in several ways been misunderstood and misapplied. We only know energy as the energy or working power of something, i. e. the ability of something to overcome a certain resistance through a certain distance. It is therefore merely fantastic to talk of energy, where no substance containing this energy is to be found. Furthermore, psychical energy has often been spoken of as a proper scientific notion; but if such a thing really exists or not, we do not as yet know. Such dogmatic extensions must therefore be kept apart.

In the field of muscular research it is first of all the conception of work, which has given rise to the confusion. To keep your arm extended at a level for twenty minutes is without gainsaying a hard piece of work — in an ordinary popular sense of the term. But from a physical point of view it is no work at all, as the resistance in this case is not overcome through any distance. It is therefore not allowable to introduce into the doctrine of energy the new notion of *statical work*, representing the overcoming of resistance at rest. This would create contradiction in the doctrine of energy and quite destroy it. It is quite true that chemical energy is consumed in an experiment like the above mentioned, but if the muscular action is to be explained in concordance with the law of energy, it is here out of the question to speak about work. We have to look out for other means, and, as will be shown, such are not difficult to find.

We will therefore in the following try how far it is possible, by using the ordinary law of energy, to penetrate to a more intimate knowledge of the laws of organic muscular action.

II.

Let us commence with a mental experiment: I hold in my hand a load $p = mg$ and lift it with the constant speed

c, the distance h perpendicularly upwards, bring it to rest, and lower it again with the same constant speed to the starting point. Let us use Kilogram pressure, Meters and Seconds as units, say $p = 10$, $c = 0{,}5$, $h = 0{,}6$ and, not considering the motion of the arm, only regard the purely physical part of the experiment. We might then e. g. say: I have used the force 10 both in raising and in lowering the load. By raising it I gave it a surplus of potential energy = 6 kgm, and deprived it of the same quantity of energy by lowering it. This is also, on the whole, true, but considering the following, we are bound to investigate the question a little closer. Properly speaking, I must begin by raising the velocity 0 to the velocity 0,5. I have therefore during the first moments to increase the force 10 by a certain quantity, which must again disappear when the velocity has become 0,5, and a similar deduction has to be made from the force 10 in order to arrest the motion at the height h above the starting point. In the same way the lowering has to be started by a certain deduction from and stopped by a certain increase of the force 10.

This may be done in several different ways, but suppose e. g. we let each of the two positive excesses increase from 0 and again decrease to 0 according to a parabolic curve, in such a way that the constant velocity, c, is attained during 0,1 second, and arrange the two negative excesses in a similar way, then, as may easily be calculated, the excess of force at the utmost will be 7,645 kg and the velocity, c, will be reached 2,5 cm from the starting point and therefore also stop 2,5 cm from the end of the path, just as the two negative excesses will be equal to the two positive ones taken with opposite signs. The excesses will therefore, as well as a total, as for each of the motions, neutralize each other, and the same will be the case with the excesses of energy of which each of the positive ones will be 0,127 kgm. This last neutralizing was also to be expected on account of the law of energy, as we have evidently, by raising it, just given the load the surplus energy of 6 kgm, and again, by lowering it, just deprived it of the same surplus.

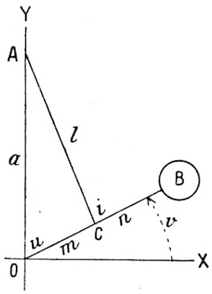
But this shows that, properly speaking, it is quite impossible to perform such raisings and lowerings with wholly constant velocity. It is quite true, in the mechanic world it may be arranged in such a way that the said excesses of force and energy just neutralize each other, but, as will be shown, it is a different matter whether the same holds good in the organic world. It is therefore important to keep in mind these circumstances.

Altogether we may, even here, notice that something rather mysterious appears as soon as we pass from the external physical to the organic field. By raising the load I have given it the energy surplus of 6 kgm. But, during the whole raising I have also spent an average lifting force of 10 kgm, and I have felt a certain strain in the action. During the lowering I have, on the contrary, deprived the load of the same amount of energy. And yet through the same distance I have spent the same lifting force and also felt exhaustion. I have no perception of being the richer in energy for this last exhaustion, quite the contrary. But what, then, has become of the energy taken away from the load?

III.

For further enlightenment another brief mental experiment must be made. Suppose OB to be a weightless lever which can rotate upon O in the perpendicular plane XOY. At the distance $OB = n$ from O a load, B, is attached to it, while CA represents a muscle which by contracting rotates OB at the constant angular velocity ω from $\angle v = -v_1$ to $+v_1$. $OA = a$ is firm. The muscle has the arm $OC = m$, the length

l and the tension p. Suppose that the said angular velocity has already been attained by $v = -v_1$ and that it does not cease till v has become $+v_1$, and let B be 10 kg, $a = 30$ cm, $m = 3$ cm, $n = 30$ cm, and $v_1 = 75°$.



If the constant velocity is already attained by $-v_1$ and maintained until $+v_1$ we are able to calculate the movement between these two limits, only determining the two opposite turning moments as equal in strength. The moment $Bn \cos v$ is trying to diminish $\angle v$ while $pm \sin i$ is increasing it. Now we have $l \sin i = a \sin u = a \cos v$, therefore $\sin i = \frac{a}{l} \cos v$. Thus we get

$$Bn \cos v = pm \frac{a}{l} \cos v,$$

therefore

$$p = \frac{n}{m} \frac{l}{a} B = \frac{n}{m} \frac{B}{a} l = \frac{10}{3} l \qquad (1)$$

and

$$l = \sqrt{a^2 + m^2 - 2am \sin v} = \sqrt{909 - 180 \sin v}. \qquad (2)$$

The energy which the muscle has expended by lifting B is

$$E = \int_{-v_1}^{v_1} p \sin i \, m \, dv = \int_{-v_1}^{v_1} pm \frac{a}{l} \cos v \, dv = \int_{-v_1}^{v_1} Bn \cos v \, dv$$

$$= 2Bn \sin 75° = 5{,}79558 \text{ kgm}, \qquad (3)$$

and we get

$v =$	$-75°$	$-60°$	$-30°$	$0°$	$30°$	$60°$	$75°$
$l =$	32,907	32,633	31,607	30,150	28,618	27,443	27,113
$p =$	109,690	108,777	105,357	100,500	95,393	91,477	90,377

This table shows that the muscle has been contracted from ca. 33 to ca. 27 cm, and from (3) it follows that B has been raised $2n \sin 75° = 57{,}9558$, or about 58 cm. The muscle has therefore, according to (3), given off just the amount of energy which B has received as potential energy. As $\frac{a}{l}$ on an average is $= 1$, it will furthermore be seen from the second right hand expression in (3), which may also be written

$$dE = pm\frac{a}{l}d \cdot \sin v, \tag{4}$$

that it is very nearly as if the muscle had all the time been using its total tension in lifting B, but had in return lifted exactly vertically upwards in the direction of $\sin v$. As the arm n of B is just 10 times the length of the arm m of the muscle, p is also on an average about 10 times as great as B; but from the figures in the table it may further be seen, that still p has varied from ca. 110 to ca. 90 kg, at the same time as the length of the muscle has decreased from about 33 to about 27 cm, and from (1) it may furthermore be seen, that p has all the time been exactly $= \frac{10}{3} l$. To this circumstance so important to the organism we will later on return.

If B is again lowered with the same constant speed, we get results exactly as before. The muscle must use the same force through the same distance. But the load is deprived of its gained potential energy, and it is easy enough to say that this has been transferred to the muscle, but as yet that sounds rather enigmatical.

Without attacking this question yet we will proceed another step in supposing AO to be the upper arm of a man, OB the lower arm, the muscle AC representing the flexors of his arm. Let his hand and arm be armoured with a stiff and heavy glove in order that OB may get exactly the same turning moment as before, and let the glove be arranged in such a way that he can keep his wrist and fingers quite unmoved. The elbow O is supported in order that the upper arm may not swing back during the motion; in short: we imagine all kinds of precautions taken in order that he may keep all the rest of his muscles from assisting at the lifting and lowering of his lower arm.

Of cause it will easily be seen that such an isolation of a particular muscle or motion is in reality quite out of the question, and on this depends mainly the difficulty of the problem raised. Do what we will, there is no doubt that the man's breathing and the action of his heart must still play their part during the experiment. Nor will he perhaps be able to keep the extensors of his arm quite inactive during the experiment. His upper arm will have to carry the increased weight of the forearm, and this again will probably cause several other muscular tensions as well around the shoulder joint as on the opposite side of the body for the sake of equilibrium.

We are therefore *in reality* unable to establish complete isolation. But, as will be shown later on, we may at least endeavour to reach the utmost possible isolation, and even this will be of great importance. We are, however, as yet only in the field of mental experiment; but this, too, may be a road to insight; and in the field of mental experiment we *can* produce complete isolation.

We will therefore suppose all more or less irrelevant circumstances to be kept apart, and then ask: What quantity of energy has the muscle spent in raising the arm the said distance? and how is it about the lowering?

And if the law of energy is really to hold good for the organism, then the reply to this question will not be difficult.

As, by being raised, the arm has received the quantity of energy 5,79558 kgm, the muscle must have given off just the same quantity of energy and neither more nor less for this purpose. And if by being lowered the arm has lost the quantity of energy 5,79558 kgm, then the muscle — in some way or other — must have received just the same quantity. Otherwise the validity of the law of energy for the organism must be given up.

IV.

Let us, then, examine what will be the consequences of these considerations.

From a scientific point of view we cannot justly speak of *statical muscular work*; but there is nothing to prevent us from using the term *statical muscular action.*

Suppose then the said loaded forearm be kept a certain time, t, in a certain elevated position. We do not consider how it has got into this position, neither do we care for any but the named circumstance. No work has then been performed, and the quantity of energy spent on the experiment must therefore be $= 0$.

Of course it is right that in any such experiment a certain quantity of energy is *transformed*, and this transformation is, in the last instance, a transformation into heat. But the law of energy demands that the *consumption* of energy, the *expenditure* of energy, be $= 0$. If K be the lost *chemical* energy and V the heat gained, we therefore — both quantities being measured in mechanical units — get

$$V = K.$$

In conformity with all our daily experiences it will further be extremely reasonable to suppose the transformation proportional to the muscular tension, p, the time it has lasted, t, and a coefficient, c, which must be dependent — amongst other things — on the extent to which the muscle has been contracted during the experiment. The more contracted the muscle is, the greater will be the exertion required to produce a certain tension; and that this feeling of exertion is not quite a delusion, we have sufficient evidence of. Of course these suppositions are as yet but hypothetical and require to be further examined; but if they are correct, and if we term the muscle expenditure of energy during the experiment U_0, we may write:

$$U_0 = cpt - V = 0, \qquad V = cpt. \tag{5}$$

Let the muscle have had the length l_2 during the experiment, and now suppose that it performs a certain amount of work, A, in contracting itself from l_3 to l_1. Let these limits be arranged in such a way on both sides of l_2 that we may assume an average muscular tension, p, as before, and also an average coefficient, c, as before. The time for the experiment may also be assumed unaltered. We are then justified in supposing that the same average tension, during the same time and by the same average contraction as before, has required the same chemical transformation as before. We cannot, however, simply repeat the equation (5), the law of energy now demanding a real expenditure of energy, A, from the organism. We may, however, presume either

$$U_1 = cpt - (V - A) = A, \qquad V = cpt \qquad (6)$$

or

$$U_1 = (cpt + A) - V = A, \qquad V = cpt. \qquad (7)$$

In the first case the chemical expenditure is as before, but the quantity of heat A kgm less. In the latter case the production of heat is as before, but the chemical expenditure A kgm more than before. As will be shown later on, there is good reason to prefer the last supposition.

We will, however, first examine the third instance: The muscle extends itself from l_1 to l_3 thus lowering the arm which thereby loses the energy A. According to the law of energy the total expenditure of the muscle must then be negative, the muscle must become enriched by the amount of energy which the arm has lost, and according to the preceding we may therefore write either

$$U_2 = (cpt - A) - V = -A, \qquad V = cpt \qquad (8)$$

or

$$U_2 = cpt - (V + A) = -A, \qquad V = cpt. \qquad (9)$$

We are here again confronted with the mysterious problem: How can the outer world send energy to the muscle? There is no heat conductor from the lowered load to the interior of the organism, neither has the load been deprived of heat.

Before we proceed to answer this question, we have first to decide which of the different suppositions set forth in the four last equations are the most valuable, and, for reasons which will be given below, we shall be justified in preferring the equations (5), (7) and (9), whereby, in the three instances, we get

$$\left.\begin{aligned} U_0 &= cpt - V &&= 0 \\ U_1 &= (cpt + A) - V &&= A \\ U_2 &= cpt - (A + V) &&= -A \end{aligned}\right\} V = cpt, \qquad (10)$$

where the brackets indicate that, in the second instance the chemical expenditure is $cpt + A$, and in the last instance the production of heat $= A + V$.

For preliminary confirmation this: Already by bringing our rough experiences from daily life into connection with the law of energy do we get such important information about the conditions of our muscular action that we are not far from at once being able to formulate the equations just given. We cannot long avoid discovering that in all muscular work something like a double expenditure may be observed, or rather two expenditures, relatively independent of each other, two separate functions, mathematically speaking, a pronounced *function of time* and a special *function of work.*

Suppose I perform a certain piece of work: e. g. raise myself in a bar, then I always, howsoever I may perform the feat, get a feeling that it is so and so hard a piece of work, far more difficult, for example, than to lift an arm or such like, and the law of energy furthermore convinces me that I have, of course, to pay the proper price which the work demands.

But, furthermore, I soon find out that I can perform the exercise in such different ways that the difficulty of doing it assumes quite different degrees. If I spend 16 seconds in raising myself, it is much more difficult than if I do the work during 2 seconds, nay moreover, my feeling of exertion tells me quite plainly that the difficulty — at least within certain limits — always increases with the time.

If it is an easier exercise I perform, I will soon find out that, here again, the stress increases according to the time, even as the two exercises together inform me that the difficulty also depends on, how great a tension I have to give the muscles. Nay, the intelligent sportsman will furthermore gradually discover, that it is more difficult to develop a certain tension in a muscle when it is much contracted than when it has its full length. It is evidently far more difficult to keep yourself suspended for a certain time in bended than in almost straight arms.

It is therefore reasonable to make the function of time $= cpt$. This function appears independently in all muscular action, whether any work has been done or not. It can therefore only cause transformation of energy but no loss or gain. If, however, the law of energy is to be maintained, a function of work, A, must be connected with this function of time, and as work is only produced when the muscle contracts or extends itself, this function of work must evidently be specially connected with these variations of the length of the muscle. Now the muscle performs a piece of work every time it contracts itself. It thus gives out energy and so must itself become poorer in energy. In extending itself in the aforesaid manner, it, on the contrary, receives energy and must in so far, become richer in energy. But this is easiest and best understood in supposing that the muscle, while meeting resistance, by contracting itself a certain distance *loses* a certain equivalent of heat corresponding to the resistance and distance, while in extending itself during resistance it *gains* an equivalent of heat corresponding to the resistance and distance. For different reasons we must furthermore assume, that the loss of heat during the contraction again compels the muscle to transform an additional, corresponding amount of chemical energy into heat, in such a way that no refrigerating takes place, but the loss is purely chemical. This explains to us, amongst other things, that, after all, we feel it more difficult to raise ourselves by the arms than to lower ourselves or to rest midway. That in this way we can receive a certain enlightenment through our feeling about the economy of our muscles, is but in accordance with the apparent suitability of the whole organism.

According to these assumptions in all the three instances there will be a production of heat corresponding to the expenditure of chemical energy cpt. In the last instance, in negative muscular work, there will furthermore be a production of heat corresponding to the quantity of energy, A, which the external world has been deprived of, whilst in the second instance, in positive muscular work, transformation to heat will take place as well from the quantity of chemical energy, cpt, as from the quantity of chemical energy, A, corresponding to the loss of heat by the contraction of the muscle or to the work performed by it.

In the following we will examine the validity of these suppositions still closer.

V.

We must, however, first transform the equations given above, in order that they may be more fit for practical use.

As we only in few cases know what has taken place in the organism, while as a rule it is easy to determine what has taken place in the external world, it would be an advantage if, everywhere in the three formulas, we could change the internal quantities for external ones.

With regard to the muscular work, A, it is easy enough. According to the law of energy and to our example above, this work must be put equal to the work done in the external world. Very often this only consists in the lifting or lowering of a certain load along or against the direction of gravity. If the load is B and the distance s, we may then write

$$\pm A = \pm Bs,$$

nor, if the external work has another shape, will the transformation in that case cause any difficulty.

More trouble is given by the quantity cpt. The time, t, may, however, easily be determined.

For the tension, p, of the muscle we found in our former example the equation

$$p = \frac{n}{m}\frac{l}{a}B, \tag{11}$$

and the same shape will be assumed by the relation between p and B in a number of elementary cases where the pull of the muscle lies in the plane of the motion, to which cases we will here confine ourselves.

In the said formula m and n are simply the two levers, viz. that of the muscle and that of the load. Two corresponding levers will be found in most cases of muscular work; but the ratio between them may of course be very different, and often it will be necessary to directly determine it, before we can proceed.

Still more trouble is given by the ratio $\frac{l}{a}$ or the ratio between the length of the muscle in each moment and the distance between its point of origin and the axis of rotation. Not even in the single case is this ratio constant, as during the motion the muscle contracts or extends itself. As a first approximation it may be put $= 1$; but if a more exact calculation is required, this ratio has also to be determined in each case.

As l decreases as the contraction proceeds, then, according to (11), B being constant, p will also decrease. This is a fairly general law for all muscular work and a very profitable circumstance for the organism as, in fact, a certain tension becomes more and more difficult to produce, the more the muscle is contracted. In the calculation of the muscular action, however, this circumstance, on the contrary, is a further difficulty on the road.

We have still left the coefficient c, about which we at present only know, that it increases with the contraction of the muscle and is probably 4 or 5 times as great by the least length of the muscle as by the greatest.

We therefore meet uncertainty and difficulties enough. But in any case we are at least capable of transforming the expression cpt in such a way that we get

$$cpt = c\frac{n}{m}\frac{l}{a}Bt = kBt, \tag{12}$$

where k is a new coefficient and B the external load. We may thus transform the three former equations to

$$\left.\begin{aligned} U_0 &= kBt \qquad\qquad - V = 0 \\ U_1 &= (kBt + Bs) - V = Bs \\ U_2 &= kBt - (Bs + V) = -Bs \end{aligned}\right\} v = kBt \tag{13}$$

The coefficient k is, however, still unknown. According to (12) it cannot be a general coefficient for the muscular action, not even if we use the approximation 1 for $\frac{l}{a}$. As long as $\frac{n}{m}$ is the same for a series of experiments, it will, however, in its variations tolerably follow c and, like this, increase with the contraction of the muscle. As it would be rather interesting, if, before we go to the further empirical investigations, we might be able to get, at least, a rough idea about what kind of quantity k is, we will attempt to get a notion about it from our daily experiences.

The difficulty, we here have to face, is, however, very great and arises mainly from the circumstance that, supposing the former assumptions to be correct, it is out of the question to speak about a determined quantity of transformation of energy corresponding to a certain external work, as the quantity of transformation will then belong as well to the work itself as to the time it has lasted. Even the least piece of work may then, as said, use as large a transformation as may be imagined, if the work is only performed at a sufficiently slow rate. When various rough experiences contrarily seem to prove, that the external work generally demands a transformation from 3 to 5 times its own value, this probably arises from the fact, that in most cases in such work a certain ratio has been prevailing between the time and the work itself, which is suitable, but by no means necessary, for the organism. It may e. g. be quite proper to use 3 or 4 seconds in lifting the fore-arm from $-75°$ to $+75°$, if it is as heavily loaded as before mentioned. A workman who for some time had to perform such a motion, would surely involuntarily choose such a speed, while an experimentalist is likely to choose various speeds, according to the various aims he has in view. We may therefore perhaps get a rough estimate with regard to what kind of quantity k is, in supposing that the said armlifting has lasted 3 seconds and in itself has demanded a transformation of 3 times the value of the external work. As the external work itself, Bs, was $= 5{,}8$ kgm, kBt will be $= 2 \times 5{,}8 = 11{,}6$, kt therefore $= 1{,}16$ and $k = \frac{1{,}16}{3} = 0{,}39$.

This is, however, only an average value of k. If greatest contraction is represented by 1 and if we assume 10 degrees with even rise, whilst at the same time we let k rise at an even rate and assume its greatest value 4,5 times as large as the lowest, we get the following series, where f denotes the contraction:

$f =$	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
$k =$	0,14	0,19	0,24	0,29	0,34	0,39	0,44	0,49	0,54	0,59	0,64.

In case we introduced the even more probable assumption that k at first increased somewhat slower and at last somewhat faster than above supposed, we might let the values of k be determined by a parabolic curve and e. g. put

$$k = 0{,}15 + 0{,}3\,f + 0{,}3\,f^2,$$

which would give

$f =$	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
$k =$	0,150	0,183	0,222	0,267	0,318	0,375	0,438	0,507	0,582	0,633	0,750

The greatest value is here supposed 5 times the least; but of course both assumptions are but quite arbitrary suppositions. It is, however, not unimportant a priori to form a conception of what we may reasonably expect to find by the intended researches. To operate with hypotheses is in itself not objectionable. It only becomes objectionable, when we forget that we have only hypotheses, and not proved assertions, before us.

VI.

We now proceed to examine what the various experimental results may teach us about the muscular action.

Amongst the many valuable experiments which have been made for the elucidation of the question, those made by J. E. Johansson partly in conjunction with G. Koraen are no doubt some of the most extensive and valuable. They were carried on during several years and are fully described in *Skandinavisches Archiv für Physiologie*, especially vol. 11, 13 and 14, 1901—1903. In the following we will make them the subject of a careful examination.

The experimenter, as a rule Professor J. himself, to whose personal experiments we will in the following confine ourselves, is placed in a chair with back support at a small table about 50 cm wide which has a metal disc fitted on the opposite side. This disc has two grooves at the circumference, and in each of them is fixed a chain, one of which carries a weight which may be raised and lowered, as the disc revolves; the other is connected to a sledge which can slide on an iron rail across the table and is fitted with two handles. The experimenter may thus lift and lower the weight up to 50 cm by taking hold of the handles and hauling the sledge from the disc towards himself or easing it off to the disc. The apparatus was fitted with various other contrivances to indicate the time and position of the sledge, to retain it at various places on the rail etc. etc. With regard to these I may be allowed to refer to the description in vol. 11 of the said periodical. The apparatus was placed in TIGERSTEDT's and SONDÉN's respiration chamber at the *Carolinska medico-chirurgiske Institut* in *Stockholm*, and it was the carbonic acid expired during each period of trial that was measured. Each period lasted for half an hour or sometimes a whole hour, and the experiments were carried out with weights of 10, 20 and 30 kg.

Two, according to my view, fortunate circumstances made these experiments specially remarkable.

Firstly the three kinds of muscular action could be examined separately. The weight could be kept at rest for a shorter or longer time, while the arms were more or less bent. It could be lifted, up to 50 cm, one time after another, as, after each lift, the sledge was automatically carried out to the disc, and it could also repeatedly be lowered, as the sledge (in this case unfortunately by the experimenter himself, however, unloaded) was carried back from the disc. In this way average results from numerous experiments could be obtained.

Secondly the large number of each kind of experiments made it possible to eliminate various elements from the total results. The following will illustrate this further.

First, however, we have to examine, how the three formulas

$$\begin{aligned} U_0 &= kBt - V = 0 \\ U_1 &= (kBt + Bs) - V = Bs \\ U_2 &= kBt - (Bs + V) = -Bs \end{aligned} \qquad (14)$$

will appear with respect to an arrangement of trials as the afore said.

The experiments take no account of the heat production; only the expired quantity of carbonic acid is noted down. If the three formulas are correct, this — the presumable extra action being set aside — must in the first instance arise from the transformation of the quantity of chemical energy kBt, in the second instance from the transformation of the quantity of chemical energy $kBt + Bs$, and in the last instance as in the first, from the transformation of the quantity of chemical energy kBt. If the said quantity of carbonic acid is calculated as energy, and if in concordance with TIGERSTEDT we put 1 g expired CO_2 equivalent to 1200 kgm, we arrive at the three energy equations

$$\begin{aligned} u_0 &= kBt \\ u_1 &= kBt + Bs \\ u_2 &= kBt \end{aligned} \tag{15}$$

where u denotes the equivalent of energy of the expired carbonic acid arising from the purposed action itself. If these formulas show themselves correct, the former must also be correct if the law of energy is to retain its validity for the organism. We now proceed to examine what the experiments made by Johansson can teach us about this.

VII.

He commences (in vol. 11) by examining the *static action.* He first assures himself whether the expenditure of carbonic acid during the trial period actually increases proportionally to the numbers of identical experiments or not. In nearly stretched arms he holds in position 10 kg during 1 second, 180 times in one half hour, 900 times in another half hour, 450 times in a third half hour, and so forth. For the first period he has an expenditure of carbonic acid of 11,9 g and for the second 15,8 g. If the expenditure during rest for half an hour is $= x$, and the addition for the action during one second is $= y$, we get:

$$x + 900\,y = 15{,}8\,\text{g}$$
$$x + 180\,y = 11{,}9\,\text{g}$$

and consequently

$$720\,y = 3{,}9\,\text{g};$$

but from this we get $y = 5{,}42$ mg, $900y = 4{,}9$ g and therefore $x = 15{,}8 - 4{,}9 = 10{,}9$ g. He makes a further number of similar experiments and then finds, by the method of the least squares, that the values $x = 10{,}75$ g and $y = 5{,}5$ mg are best in accordance with all the experiments, and that the value 10,75 g, as denoting the expenditure during rest for half an hour, is besides quite in accordance with what he has found by actually resting in the said time.

In the same way he then finds that if the weight is 20 kg, the experiment costs 8,3 mg, and if the weight is 30 kg, it costs 11,9 mg carbonic acid. His apparatus was at that time not yet quite in order, and he therefore presumes, that the expenditures for the three weights held in position during one second ought more correctly to have been 4, 8 and 12 mg carbonic acid.

He next holds in position each of the three weights during 2 seconds and then gets the carbonic acid expenditures 6,0, 9,7 and 15,1 mg. Here again he makes a correction to 5, 10, 15 mg. But then he is brought to the, at first sight, surprising result, that as the holding in position of a certain weight during 2 seconds according to all probability in itself ought to cost twice as much as the holding in position of the same weight during one second, the six actions in themselves can only have caused the six expenditures 1, 2, 3 and 2, 4, 6 mg while the remaining expenditures 3, 6, 9 and 3, 6, 9 must have been caused by *extra action*. And if this has been the same whether the experiment has lasted for one or two seconds, it surely must have been preliminary action before the proper action (and not an assistant action during the same).

That the introduction to an experiment has cost about three times as much as the proper experiment lasting a second, is only at first sight surprising. For the experimenter has not only before the experiment to bring his hands to the handles and take hold of them with a sufficiently firm grasp; but he has also to bring a number of the muscles of the body into such a tension, that he feels sure of maintaining his position and not being pulled forward or pressed harder against the table, when the weight commences to act. Moreover, we must surely suppose that some of these muscular tensions must necessarily continue during the proper experiment, and that the extra action therefore must necessarily comprise not only a considerable *preliminary action* but also a certain *assistant action* during the experiment proper for maintaining the position of the body. The above given values may therefore only be considered as a series of first approximations.

Nor does Johansson content himself with the results thus obtained. He continues his examination of the static action (in vol. 13) and, through an extensive series of experiments, varied both with regard to weight, time and the distances of the handles from the disc, i. e. the contractions of the muscles, he arrives at the result, that the expenditure by static muscular action is really proportional to the weight and time and increases with the degree of contraction of the muscles, increasing, however, somewhat faster than proportionally to the same.

Let us examine some of his figures a little closer.

He experiments amongst other things with the weight 20,4 kg[1], which he holds in position during one second at

[1] When here, and in the following, other weights than 10, 20 & 30 are mentioned, it is because J. has also included the friction etc.

different distances from the disc. Having eliminated the extra expenditure, the nature of which he does not further discuss, he obtains a series of values corresponding to the different degrees of contraction, almost from full stretch to full contraction, and by graphic interpolation he therefrom obtains the following series of values, corresponding to the distance, D, of the handles from the disc, from 0 to 50 cm with intervals of 5 cm:

D =	0	5	10	15	20	25	30	35	40	45	50 cm
K =	2,5	2,8	3,2	3,7	4,3	5,1	6,0	7,0	8,1	9,9	13,2 mg

The first row of figures corresponds to the degrees of contraction from 0 to 1; the lower row gives the carbonic acid expenditure in mg for 1 second and 20,4 kg. If we take the figures for contraction: 0,0, 0,1, 0,2 .... 1,0 as abscissæ and the carbonic acid figures as ordinates, the first half of them gives exactly a parabola with $y = 2{,}5 + 2{,}5x + 5x^2$, whilst the remainder of the ordinates increases somewhat faster, the two last terms even considerably faster than the curve demands. It might be supposed, that the reason for this alteration of law is, that the distance between the handles of the sledge, according to the drawing in vol. 11, is only a hand's breadth. If the distance between the handles had been a shoulder-breadth, the muscular action would undoubtedly have been more even and placed more favourably during the greater contractions. The main point is, however, that the experiments thus show that the expenditure of CO_2 is about 5 times as great at full contraction as at full stretch, a result in good conformity with what an immediate estimate seems to indicate.

As 2,5 mg CO_2 represents $2{,}5 \times 1{,}2 = 3$ kgm energy, we get from the formula $u_0 = kBt$ for the least value of k

$$u_0 = k \times 20{,}4 \times 1 = 3, \qquad k = 0{,}147$$

and in the same way for the whole row

$f =$	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
$k =$	0,147	0,165	0,188	0,218	0,253	0,300	0,353	0,412	0,476	0,582	0,776

If the figures are altered according to the approximate parabola

$$k = 0{,}15 + 0{,}1f + 0{,}5f^2$$

we get

$f =$	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
$k =$	0,150	0,165	0,190	0,225	0,270	0,325	0,390	0,465	0,550	0,645	0,750,

which thus follows a single law[1].

But at any rate the formula

$$u_0 = kBt \qquad \text{or} \qquad U_0 = kBt - V = 0$$

has received good empirical confirmation by the investigations.

VIII.

From the examination of statical action, J. in vol. 13 proceeds to the examination of the *negative work* which the muscles perform in lowering a weight with constant speed. Even in his first paper (in vol. 11) he has made several trials about the lowering of different weights, while time and distance were unaltered. By these trials he finds the expenditure rising approximately to the magnitude of weight. But first he discovers that, when he manipulates the largest of his weights, a certain disturbing fatigue commences to appear, and furthermore an increased extra work comes in, as after each lowering he has to pull the unloaded sledge back himself. He therefore does not seem to have been satisfied with the result obtained, and has at any rate not put it down.

In vol. 13 he therefore continues these investigations, now varying both weight, time and distance, and eliminating the extra action. His result is, that the negative muscular work

[1] The row of figures p. 18 and 19 has thus at any rate given us the order of magnitude of k correctly.

exactly, or at least very approximately, requires the same expenditure of carbonic acid as the statical action (taking for the coefficient k the average of the different values which have been in play).

By this the formula $u_2 = kBt$ has also been found correct. With regard to the formula $U_2 = kBt - (Bs + V) = -Bs$ Johansson's experiments have not, however, decided anything directly. But indirectly this is also confirmed, as the law of energy compels us to make this supplementary addition. If carbonic acid has been produced corresponding to the amount of energy kBt, and if no external work has been performed, then a quantity of heat, $V = kBt$, must have been produced, and if even an external negative work, $-Bs$, has been produced, then, according to the law of energy, a quantity of energy, Bs, must furthermore have been produced in the muscle, and this cannot very well be supposed to have appeared in any other form than as heat. In fact, we have thus likewise obtained empirical confirmation of the formulas

$$u_2 = kBt \text{ and } U_2 = kBt - (Bs + V) = -Bs.$$

IX.

It now remains to examine the expenditure of CO_2 in *positive muscular work.*

Johansson has devoted a multitude of experiments to this matter, without, however, having succeeded in throwing full light on the question. Neither is this, if the above given formula be correct, much to be wondered at; for for positive muscular work he ought then, even net, to have

$$u_1 = kBt + Bs,$$

thus, contrarily to the former, a binomial formula. And as no doubt extra action is included in each of the two terms, and this action may here doubtless be rather predominant, his constantly used method — explained above by his calculation of the expenditure at rest — cannot be employed here without easily causing the introduction of several hypotheses which may lead to results corresponding to no reality.

To make the matter more plain, I will therefore first, in using his figures, proceed in my own way, and then examine whether our results agree, and, if not, try to find out whence the differences between them arise.

We will commence by putting forth a definite supposition and test its correctness by means of Johansson's most indisputable trial results.

We then presume that the above given net formula is correct, and that the additional extra action is partly a preliminary action, F, and partly an assistant action, M, during the trial itself.

It will furthermore be reasonable to suppose F proportional to the work, Bs, which the experimenter has been told to do, and the assistant action M proportional to the weight and the time for the experiment.

If E is the total carbonic acid expenditure, K, transcribed to energy, we then get, α and β being undetermined coefficients,

$$E = Bs + kBt + \alpha Bs + \beta Bt = B[s(1 + \alpha) + t(k + \beta)], \quad (16)$$

where α and β, by showing themselves as constants, will prove the correctness of the said hypothesis.

To test if that will be so, we proceed in the following manner: We select e. g. 10 of Johansson's most reliable trial results, and thereby get 10 values for each of the quantities E, B, s and t. By using the adjusted k-series, page 25, we determine the corresponding values of k, as a mean of the quantities k which in each case have been co-operating. We may then calculate the 10 corresponding values of u_1, and may then write down 10 equations of the form

$$\frac{E - u_1}{B} = s\alpha + t\beta. \quad (17)$$

From these 10 equations we calculate α and β as constants by the method of the least squares, and with the values thus obtained for α and β we finally determine by means of (16) 10 *calculated* values of E. If these are designated E', the quantities E' and E, if the hypothesis be correct, must coincide so closely two and two that the differences may be naturally accounted for by the imperfection of the experiments.

First of all we must thus carefully select the 10 most reliable trial results.

Johansson has discussed the positive muscular action partly in vol. 11 and partly in vol. 14. We select 5 results from each volume.

In vol. 11, page 295—299, 9 results are given. We reject from these, first the three with the weights of 32 kg, as during the experiments with these weights pronounced fatigue has now and then appeared, and it may therefore be supposed that some fatigue may always have been present during these trials. And we further reject the experiment with $t = 0{,}36$ seconds, as this can hardly have had the same exactness as the others with a less minute time limitation.

Each of these trial results is the mean of about a score of single results from similar experimental periods. The adjustment by the method of the least squares is here quite justified, and the results obtained must in so far be considered as specially exact.

The 5 results from vol. 14 are all taken from table 4, p. 66, as, for different reasons which will be given later on, this may most likely be considered as one af the most reliable of the tables published in the last paper. Properly speaking, each of the 10 results ought to enter the calculation with its special "weight". But as this is actually dependent on more circumstances than those mentioned in the paper, I have confined myself to considering the 10 results as being of equal "weight".

As a result of the stated calculation we first then get by (17)

$$\alpha = 4{,}230 \qquad \beta = 0{,}238 \qquad (18)$$

and then by (16) the table below, where D is the initial distance of the handles from the disc, K the *measured* quantity of carbonic acid in mg, E its equivalent of energy in kgm and E' the quantity of energy in kgm, *calculated* from the used hypothesis. The other letters denote the same as before. The calculation is therefore made by using the equations

$$E = 1{,}2K, \; u_1 = B(s + kt), \; E' = u_1 + B(\alpha s + \beta t) \quad (19)$$

No.	D	B	s	t	K	E	k	u_1	E'	$E'-E$
1	0	21,7	0,5	1,1	59,4	71,28	0,375	19,80	71,38	+ 0,10
2	0	10,9	0,5	1,0	29,8	35,76	0,375	9,54	35,19	— 0,57
3	0	21,7	0,5	1,0	57,3	68,76	0,375	18,99	70,05	+ 1,29
4	0	21,7	0,2	0,5	25,6	30,72	0,200	6,51	27,45	— 3,27
5	0	21,7	0,2	1,4	30,0	36,00	0,200	10,42	36,00	0,00
6	0	22	0,487	0,5	53,0	63,60	0,375	14,84	62,78	— 0,82
7	0	21,7	0,500	1,1	59,0	70,80	0,375	19,80	71,38	+ 0,58
8	0	21,2	0,496	2,4	71,0	85,20	0,375	29,60	86,18	+ 0,98
9	0	21	0,495	5,4	103,0	123,60	0,375	52,92	123,88	+ 0,28
10	0	21	0,501	12,3	178,0	213,60	0,375	107,38	213,36	— 0,24

It will be noticed that, with the exception of No. 3 and 4, the difference between the computed and the measured gross energy is even remarkably small. It is almost surprising that the experimenter has with such a degree of exactness been able to adapt his muscle tensions to the amount of work demanded, and the results testify as well to Prof. Johansson's great ability as to the great exactness with which the experiments have been carried through. That the presented hypothesis with great exactness expresses the actual facts is hardly to be doubted, so much the less as it may without any difficulty be explained whence the greater deviation of the two said results arises.

For, if from the above given table we calculate the observed quantity of carbonic acid for the work 1 kgm, in the four instances 1, 2, 3 and 7, which all have the same s and the time very nearly alike, viz. 1 and 1,1 second, we get the four values 5,475, 5,468, 5,281 and 5,438 mg, thus in the three instances very nearly 5,5 mg, but in No. 3 not even 5,3 mg. An error has therefore, no doubt, taken place here either of observation, calculation or noting down, and we may no doubt be entitled to assume that No. 3 can hardly have had the expenditure for 1 kgm work say less than 5,4 mg. But if this value is used, the energy expenditure increases to $5{,}4 \times 1{,}2 \times 21{,}7 \times 0{,}5 = 70{,}31$ kgm, whereby the former large excess of E' even becomes a small deficit, 0,26 kgm. By only increasing the expenditure to 5,38 we should have struck the balance.

In No. 4 the difference between E' and E is even far greater. The calculated gross expenditure is only about $^9/_{10}$ of the observed. But we have also here an instance which really ought not to have been included amongst the specially exact. For s is here only 0,2 and t only 0,5. It is thus demanded that the experimenter lifts the weight only for the short distance of 20 cm, and this he must take care to perform during exactly half a second, if possible with constant speed through the greatest part of the distance. But this is a very severe demand on the experimenter's ability, surely a more severe demand than any one can accomplish with essential approximation. A considerable inexactness is likely to take place both with regard to distance and time, and even if one would be able to estimate the errors committed and allow for them according to some valuable hypothesis, it may not be easy to allow for the increased muscular tension, which the experimenter has no doubt instinctively introduced into his attempt to accomplish the severe demands as carefully as possible. It may therefore with great probability be expected that in all such minute experiments, the equivalent of energy of the observed carbonic acid will appear considerably larger than the quantity of energy which is obtained by calculation according to the formulas, which have proved to be very satisfactory in the more practicable experiments. A few examples will sufficiently confirm this.

In vol. 14 p. 71 are mentioned some experiments with the weight 21,7 kg out of which we will choose the first of each kind for further elucidation. It is the initial position of the handles that is here varied. While their initial distance, D, from the disc has up till now been 0, it is here respectively 0, 10, 20, 30, 40 cm. This is, however, not the specially determining fact. But s is in each case only about 10 cm and t only about ¼ second. The circumstances are thus far more difficult than in number 4, and it is therefore not to be wondered at that the ratio between E' and E, by a calculation exactly similar to the one before, becomes still more unfavourable than in number 4. We get

No.	D	B	s	t	K	E	k	u_1	E'	$E'-E$
11	0	21,7	0,099	0,25	14,5	17,40	0,168	3,0597	13,4382	—3,9618
12	10	21,7	0,100	0,26	15,4	18,48	0,228	3,4564	13,9783	—4,5017
13	20	21,7	0,102	0,22	15,2	18,24	0,328	3,7793	14,2782	—3,9618
14	30	21,7	0,102	0,25	16,7	20,04	0,468	4,7523	15,4061	—4,6339
15	40	21,7	0,102	0,25	18,6	22,32	0,648	5,7288	16,3826	—5,9374

While the calculated expenditure here lies between 13,4 and 16,4 kgm, the observed expenditure lies between 17,4 and 22,3 kgm. The latter has thus an excess of 30 p. ct. or more, and it is hardly to be supposed that this large excess at E has any other origin than special muscular tensions originating in the solicitude to perform the difficult experiments as precisely as possible. That this excess should follow any simple law is hardly to be expected and it will therefore no doubt be most practical and methodical for the present to neglect all such minute experiments and first try to master the more simple ones.

Let us now look a little closer at Johansson's own proceedings.

As said, he commences the examination of positive muscular action in vol. 11. He subtracts the expenditure at rest, in the formerly described way, and then finds, that the remaining expenditure of CO_2 increases fairly approximately in proportion to the outer work; for 1 kgm he thus gets a fairly constant quantity. Several experiments with the weight 32 kg have, however, caused fatigue and required a somewhat greater expenditure, especially if during a single period of trials many such experiments have been made. The expenditure for 1 kgm work increases furthermore to a certain degree with the time. Between the proper work and the extra action he has here made no separation.

In vol. 14 he tries to pass beyond these vague results. He endeavours now to find the influence of the several variables on the expenditure. These variables are D, B, s, t, and k, of which k, however, according to the above stated is determined by D and s. If the expenditure at rest be subtracted and the remaining expenditure transcribed to energy, we may therefore write

$$E = f(D, B, s, t, k). \tag{20}$$

But the functional form is still unknown, and certain precautions are therefore necessary. If we exclude the minute experiments and suppose the before given formula

$$E = B[s(1 + \alpha) + t(k + \beta)] \tag{16}$$

valid for the others, certain proceedings are, as may be seen, allowed, but others not.

Johansson has now tried to procure several series of trial results, arranged in such a way that in each series only one of the variables alters its value while the others remain constant. To this end he has transformed the directly obtained rows which as a rule had several changing variables. He has thus altered the above named table 4 in such a way that it always gets $B = 20$ kg and $s = 0{,}5$ m. Very carefully he has in this reduction taken into account that the friction during the motion varies somewhat with the speed, and with this in view he has increased or decreased K in proportion to B. Thereby he has, however, introduced the hypothesis, that K, and consequently E, also *is* proportionate to B, and this agrees with (16).

But also s is altered in the new table 5. It does not appear from Johansson's paper how great an influence on E he has attributed to this alteration of s, and it is a little difficult to see it from his rounded figures. Supposing (16) to be right only the one member of E increases proportionally to s, while the other remains unaltered. But J. has not at all used any such guiding formula. It is therefore hardly probable that he has used any other hypothesis here than the most simple, that E is proportionate also to s, thus, that in the case of s being doubled, E will also be doubled. If (16) is right this, however, must lead us astray. Even if only smaller alterations of s and similar variables are required, the indicated proceeding may produce considerable errors in E. If (16) or even a similar formula is correct, and if t is so large that the last member of E becomes greater than the first, a multiplication of s by c will cause that the whole of E becomes multiplied by c whilst not even half of it ought to have been so.

But whether Johansson now really has pursued this course or any other in a similar way arbitrarily chosen, it is in any case certain that by the several following reductions of his tables he has been led to introduce considerable errors into his figures. This may already be felt in the transformation of table 4 to table 5, and the mistakes will, as may be seen, become still greater in the following.

Having thus transformed table 4 to table 5, which throughout has $B = 20$ and $s = 0{,}5$, whereby t becomes the only changing variable, Johansson finds, by the method of the least squares, that the respective expenditures of energy, E, may be dissolved in

$$E = m + nt \tag{21}$$

where[1] m and n (for $B = 20$ and $s = 0{,}5$) are constants: $m = 43{,}4 \times 1{,}2 = 52{,}08$ and $n = 9{,}9 \times 1{,}2 = 11{,}88$ kgm. Even he has thus found that the gross energy for positive muscular work contains two members, one independent of time and one (under certain circumstances) proportional to the time. According to formula (16) we have

$$\begin{aligned} m &= Bs(1 + \alpha) = 20 \times 0{,}5 \times 5{,}230 = 52{,}30 \\ n &= B(k + \beta) = 20(0{,}375 + 0{,}238) = 12{,}26. \end{aligned} \tag{22}$$

The difference between these and Johansson's values for m and n arises no doubt partly from the reduction of table 4, but partly also from the fact that the values are derived from different starting points.

Johansson expresses (page 67) his result in the following manner: m is the expenditure for the work taken as momentary, n is the surplus for each second it has lasted.

In the remaining part of Johansson's paper the influence of the said table reductions becomes very predominant. Furthermore the results of the minute experiments are here often mixed with the more reliable results, in order to obtain the desired tables. These two circumstances cause the obtained figures to become of a doubtful value. To justify this statement we must follow the author also through his concluding remarks.

On page 75 he arrives at the result that both m an n increase proportionally to B, and in passing on to p. 76 he furthermore points out that m also increases proportionally to s. It is, however, to be noted that the expression used does not signify

[1] J. uses the letters s and t for these two quantities.

"is in proportion to"; for still on p. 76 it is said that we have[1]

$$m = m_1 + m_2 Bs. \tag{23}$$

Thus it is only the one member of m which according to Johansson is proportional to Bs, m_2 being a constant.

Now we have just found that, according to a series of specially reliable experiments, $m = Bs\,(1 + \alpha) = 5{,}230\,Bs$. This law did not, however, hold good with regard to the difficult minute experiments. There the observed E was always considerably larger than the E calculated according to the formula $E' = Bs(1 + \alpha) + Bt(k + \beta)$, and the excess has no doubt increased as well the first as the last member. Johansson has, however, arrived at his formula for m by using reliable and very minute experiments together (table 15), and as the expenditures in the minute experiments may no doubt be rather arbitrary, it is not to be wondered at that he has found m's proportionality to Bs imperfect. Possibly it is not after all absolutely perfect. But it is at any rate, with regard to those experiments that, according to all probability, may be considered most reliable, of such considerable approximation that it would be objectionable to efface this result by introducing the minute experiments.

And to this comes that both Johansson's numerical result and his interpretation of it may be quite untenable. He gets (p. 76 table 15) $m_1 = 5$ mg CO_2 $= 6$ kgm energy and $m_2 = 3{,}83$ mg CO_2 $= 4{,}596$ kgm energy, and he says: m_2 is the expenditure for 1 kgm external work performed momentarily, while m_1 is the expenditure independent of s representing the preliminary action. Now, in table 15, B is always $= 20$ kg and $m_1 = 6$ kgm, whilst on p. 76, it is stated, that for $B = 10$ kg, $m_1 = 2{,}5$ mg CO_2 $= 3$ kgm energy. From this and from other remarks it may be concluded, that m_1 is really proportional to B, viz. $= cB$, where c is a constant. As m_2 besides

[1] For the following equation J. uses the designation $s = w + vA$.

is a constant we get $m = cB + m_2Bs$. But by this m's proportionality to B also becomes effaced. It is only the one member of m that is proportional to B and it is only the other member of m that is proportional to Bs. We have thus departed still further from formula (16).

We have, however, at the same time departed from the law of energy. For if m_1 is the expenditure for the preliminary action and m_2 the expenditure per kgm for the proper work regarded as momentary, while the whole remaining expenditure of CO_2 varies with t which may at pleasure be made large or small, the law of energy can only remain valid for the organism if the whole member containing t is only a sham expenditure compensated for by heat, and *if m_2 itself is just = 1 kgm energy, representing 0,833 mg CO_2*. If the law of energy is to hold good, the figure 3,83 is between 4 and 5 times too large.

This error in the calculation can only come from the reductions and the minute experiments.

From the quantity m Johansson proceeds to the quantity n in the equation (21). According to (16) $n = B(k + \beta)$.

J. finds on p. 77, that n increases in proportion to B. This agrees with (16). At the bottom of p. 77 he furthermore finds that n "increases with" s. This also agrees with (16) as k increases with s. Finally it is said that n has no plain proportion to the external work; on the contrary n is dependent on the position of the arms during the motion. This is also in conformity with (16), the last remark in so far as k increases for a certain distance the more the arms are bent. Thus there is the best qualitative accordance between the expression for n in (16) and all Johansson's remarks about the nature of this quantity, and had he not been led astray by the reductions and the doubtful minute experiments, he would certainly, by further investigations concerning m, have found, that this quantity not only within table 5 but upon the whole within the domain of all the more reliable experiments, is really very nearly, or even in a surprising degree nearly, proportionate to Bs. And he himself would then not have been far from likevise arriving at the result

$$E = B\,[s(1+\alpha) + t(k+\beta)].$$

X.

I have here set forth what I believe may most safely be inferred from Professor Johansson's deserving and in so many respects valuable work, and I am of the opinion that the given formulas have, by the investigation here performed, after all got their empirical confirmation. By this, however, only little has been obtained regarding the muscular action, and still the obtained results require a more scrupulous elucidation.

One result, however, has at once to be fixed as a consequence of the contemplations made: We can hardly ever expect to find a single general formula for the expenditure of the organism in performing a certain piece of external work. Such a "normal ratio" does not exist. It is already rendered impossible by the extra action which no doubt always changes with the circumstances, and which will never entirely fail to appear. If we pull a sledge which at a certain speed makes the resistance m, the necessary mechanical quantity of energy for a unit of distance will, in the most elementary case, at the said speed, always be m, whether the rope is long or short, elastic or unelastic; nay, it might even be composed of a score of light spring balances, which would then all during the motion point at m, without the expenditure thereby being increased. But quite a different result will appear in the organic world. If a man is placed in a chair with his back straight up and, as Professor Johansson, pulls a 20 kg weight towards himself, then a great number of the muscles in his body must, if his position is to remain unaltered, be brought into tension like the spring balances before mentioned. But, not considering the heat relations, all these tensions involve extra expenditures, extra expenditures even if the smallest motions are avoided, which, however, may be after all impossible. The extra expenditure will therefore appear, more or less considerable, according to the special circumstances. As is well known, the supposition has been made that the expenditure by work performed with the arms is as a rule larger than the expenditure by similar work performed with the legs. This supposition is probably right, and the difference arises no doubt from the circumstance that as a rule more extra tensions will occur by work performed with the arms than by such performed with the legs, on account of the open shoulder ring etc.

Even if in course of time we should be able to get so far that in some way or other we might exactly eliminate the extra action, we could not expect to get a single net formula for performed work; for even if the formula $u_1 = B(s + kt)$ should be quite exact, k would be different in the several cases and, mind well, not only varying with the degree of contraction of the muscles but also varying with the above given ratios $\frac{n}{m}$ and $\frac{l}{a}$. Even the more fundamental coefficient, c, which (see p. 17) is contained in k, might possibly vary from one person to another, or from muscle to muscle. And to this comes that it is still uncertain what is really to be understood by the degree of contraction. Is greatest contraction to be taken as the greatest *obtainable contraction in the body* or is it something quite different? And how far is the law of contraction which agrees approximately with J.'s experiments general or not?

We have further to take note of the circumstance that the given laws confine themselves to certain simple motions with constant speed and resistance. But there exists a multitude of more compound and complicated motions. Can the laws for these be deduced from the given laws or do quite new conditions come in? What happens e. g. if a muscle after performed work, of its own accord, so to speak, returns to its ordinary length? And what happens if from that length, of its own accord, it contracts itself, in order to commence a piece of work? About this we know hardly anything.

It was furthermore pointed out in the beginning, that a simple lifting or lowering motion, can in reality never be performed quite constantly. From rest we arrive only at the constant speed by a certain acceleration, and back to rest we only arrive by some negative acceleration. In the mechanical field both the force excesses and the energy excesses at start and stopping can, as was shown, be arranged so as just to balance each other. But can the same be done with regard to the expenditures for the muscular actions? With regard to the *expenditure* of energy itself, we are, if the law of energy is to remain true for the organism, entitled to assume this. But with regard to the *transformation* of energy it may be quite a different matter.

By each lifting, at the start, that is, while the muscle is comparatively long, there has to be rendered a certain excess of force, while at stopping, that is, while the muscle is comparatively short, a corresponding deficit is required. These two therefore do not seem to be able to balance each other. At each lowering a deficit is also required while the muscle is short, and a corresponding excess while it is long. Neither in this case does there seem any probability of balance. On account of the singular relation between the contraction and the tension of the muscle mentioned p. 9 and p. 16, each case will, however, best be examined separately, and in all more exact calculations, there will thus also here be an occasion for special consideration.

Yet a single other remark I should like to add. I have not specially dwelt on the extra action for statical and negative muscular action, as I did not think, that there were data enough for a special formula to be deduced.

Johansson commences his investigation of statical action by estimating the gross expenditure for respectively 10, 20, 30 kg through 1 and 2 seconds at

4 8 12   5 10 15 mg CO_2

and the net expenditure at

1 2 3   2 4 6 - -

The extra action therefore becomes quite equal for both periods, viz. 3, 6, 9 mg. We cannot, however, acquiesce in this assumption. As several times before pointed out, there must surely have been extra action as well before as during the experiment, and the assistant action must evidently increase with the time, be it even so slight that it can hardly be traced. It is therefore to be supposed, that the gross expenditure may be represented by a formula like

$$E = B[a + (b + k)t],$$

while the net expenditure has been $u_0 = kBt$. The above stated net expenditures are, besides, according to the later experiments rather small. If we put them — expressed in energy — according to p. 24—25 at 1,5 etc. and if we put $a = 0,3$, $b = 0,02$ and k as before $= 0,15$, we get:

$u_0 =$	1,5	3	4,5	3	6	9
$E =$	4,7	9,4	14,1	6,4	12,8	19,2
$E - u_0 =$	3,2	6,4	9,6	3,4	6,8	10,2

Johansson has

$E =$	4,8	9,6	14,4	6	12	18.

The difference is thus not exceedingly great. Of course the stated formula is a mere possibility; but that the proper formula must be of some such shape, is, no doubt, most probable, just as it may also be supposed that if it really contains two constants as the indicated, these must be rather different from the two constants α and β in the gross formula for positive muscular action. Also for negative muscular work a similar formula must surely be composed.

It will be seen there are still plenty of problems left. I shall conclude with a few remarks concerning how, according to my view, they may best be attacked.

From all that has been stated it will be seen, that it is first of all the considerable extra action which bars the road to the arriving at a more precise determination of the expenditures for the proper action. Just as it will be best to exclude the probably fatiguing and the too minute experiments, and at any rate for the present restrict ourselves to the more practicable, it will also be best to try to select these in such a way that the extra action becomes as small and as well defined as possible. Several recent experimenters have used the favourite instrument of our day, the cycle, as a trial apparatus; but the said draw-back is here without doubt rather prominent. It is therefore hardly probable, that we shall make any advance by using this machine. We are more likely to succeed by less complicated and more computable experiments, as e. g. by the before mentioned motions of the fore arm, where the load is fixed to the whole fore arm, in such a way that all co-operation of wrist and fingers may be excluded, and where the elbow may be supported as well with regard to back motion as to lowering. Likewise, we might possibly advantageously use a heavy wheel, turned with the foot like a common spinning wheel. The heel might remain nearly stationary and with some practise we might manage it so that it was always the inertia of the wheel that lifted the point of the foot, whereby the treading was only performed by the muscles of the calf. The working leg would be lifted a little at each turn but in a passive way and also computably.

The operator might otherwise be comfortably resting in an easy chair during the experiment, and the wheel might in several simple and easily computable ways be braked more or less, as the times of revolution and experiment might be varied at pleasure.

It is quite true, the three kinds of muscular actions could not by these experiments be kept so completely apart from each other as by Johansson's experiments; but by performing the arm movements with different speed for lifting and lowering, we might surely overcome this difficulty.

It would of course also be an advantage if the measuring of the expenditure was performed as perfectly as possible. According to more recent experiences it would be more reliable to measure the oxygen than the carbonic acid, and still more advantageous to measure both and, if possible, also the heat produced. It would then be possible, both with regard to the extra action and the proper work, to separate the expenditure of energy and the transformation of energy.

It would be very desirable if some physiologist would carry out a number of such or similar experiments.

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